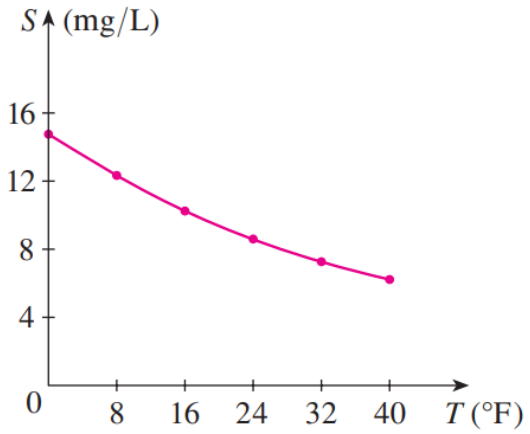
Exercise 57

The quantity of oxygen that can dissolve in water depends on the temperature of the water. (So thermal pollution influences the oxygen content of water.) The graph shows how oxygen solubility S varies as a function of the water temperature T.

(a) What is the meaning of the derivative $S'(T)$? What are its units?

(b) Estimate the value of $S'(16)$ and interpret it.



Source: C. Kupchella et al., *Environmental Science: Living Within the System of Nature*, 2d ed. (Boston: Allyn and Bacon, 1989).

Solution

(a) $S'(T)$ is the rate that the solubility increases with respect to temperature. Its units are mg/mL per Fahrenheit degree.

(b) $S'(16)$ is the rate that the solubility increases with respect to temperature when the temperature is 16 °F.

Compute the slope of the secant line from $[8, 16]$

$$\frac{S(16) - S(8)}{16 - 8} \approx \frac{10 - 12}{8} = -\frac{1}{4}$$

and the slope of the secant line from $[16, 24]$.

$$\frac{S(24) - S(16)}{24 - 16} \approx \frac{8 - 10}{8} = -\frac{1}{4}$$

Then take the average of the two to get the best estimate for S' when the temperature is 16 °F.

$$\frac{\left(-\frac{1}{4}\right) + \left(-\frac{1}{4}\right)}{2} = -\frac{1}{4} = -0.25$$

Therefore, the solubility is decreasing by 0.25 mg/mL per Fahrenheit degree when the temperature is 16 °F.